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PRELIMINARY INVESTIGATION INTO THE IDENTIFICATION OF A DISTINCTIVE MATHEMATICALLY EXTREMELY ABLE SUB-COHORT IN GCSE MATHEMATICS

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Note: Because there is no consensus in the academic literature I will, for the most part, use terms such as 'exceptionally/highly able', 'gifted' and 'talented' in the way that the researcher being reported uses them. Sometimes the writer makes the meanings and distinctions explicit, but otherwise I mean no distinction myself.

1 Conceptualisations of giftedness

Many of the psychologists, educators and others writing about exceptional ability do not limit themselves to mathematical or any other particular ability, even when they believe that all giftedness is domain specific. Sometimes a scan of research papers an author has published make it clear how widely or narrowly their own experience of giftedness lies, but rather than try to guess this for each person cited in this section I will simply treat together any not obviously irrelevant conceptualisations of what is usually referred to as 'giftedness' or 'talent'.

> Gifted children began to exist, as far as I can tell, in the second decade of the 20th century as a result of a confluence of sociocultural and sociopolitical factors that made the creation of the construct useful. (Borland, 2005, p3)

Despite, or perhaps because of, the short history of efforts to investigate giftedness, a wide range of models has been proposed. In this summary they are sorted into six groups arranged in roughly the chronological order of the groups' appearance. Note, however, that none of these groups has been abandoned; all of them currently have advocates within the research community and some of them still strongly influence public understanding of the issues.

In brief, the defining essences of the groups of models are:

1 a unitary cognitive trait, such as IQ;

2 domain specific cognitive traits;

3 cognitive, affective and motivational combination;

4a deliberate practice;

[4b neuroscientific and genetic evidence;]

5a developmental and synthetic models;

5b context models;

6 no model of giftedness.

1.1 The unitary concept of ability and giftedness

In the 19th and early 20th centuries, pioneering studies of 'genius' by Francis Galton in Britain and Lewis Terman in America were primarily interested in the heritability of ability. Because, from their perspective, a whole person was to be identified (or not) as a 'genius' there was no attempt to differentiate seperately heritable aspects of ability. Spearman (1904) gave this unitary concept its familiar name 'g', for general intelligence, and the business of measuring and using IQ in education, military placement and business began.

Conceptions in this group concentrate on *aptitude* or *potential*, rather than on observable intelligent behaviour; later theorists see this as a major weakness making the whole enterprise crude, inaccurate and useless. Nevertheless, the notion that there is a single dominant heritable factor that necessarily advantages some children more than others, and that may be called intelligence, is still held by some. Recent, and controversial, advocates of this view include Herrnstein & Murray (1994), Jensen (1998), and Brand (1996) who claims evidence from neuropsychology to support his view.

Although there seems to be very little support for this view in the academic community of giftedness researchers it is still highly influential in the public mind. In his review of NAGTY's admissions process, Strand (2006) reports, from the *Excellence in Cities* project, the conclusion that "there is also a general consensus across secondary schools that gifted pupils are those academically able across a range of subjects" (Pocklington, Fetcher-Campbell & Kendall, 2002). The truth of this consensus is irrelevant if he is correct in stating that "there is now a clear and explicit direction for NAGTY to create a National Register of the top 5% of students as a key component of national policy" (p 17). When each child is to be labeled either as Gifted or as Not-Gifted, in an all-or-nothing way, then this model is being assumed.

1.2 domain specific cognitive traits

Although he 'invented' *g*, Charles Spearman actually proposed a hierarchic model of intelligence in which *g* was the overlap between two or more 'group factors'. Louis Thurstone (1938) proposed a model with 7 cognitive abilities that jointly constituted *g*. JB Carroll's (1993) hierarchical model is a modern version of Spearman's, and is used by many researchers. Other psychologists, such as J P Guilford (1967) denied the reality of *g* and proposed a system that would create 180 discrete kinds of intelligence (though tests constructed to measure them proved to be quite highly intercorrelated). Godfrey Thomson (1939) showed that statistical analysis alone could not decide the matter of the existence or non-existence of *g*.

Several modern theories present multiple ability models, and in all of them at least one trait is particularly relevant to maths. Some refuse to accept that *g* is anything more than a statistical concept: most notably, Howard Gardner (1983/94) claimed that evidence from case studies, psychometrics, and – crucially – neurological scanning and lesion studies – supported the existence of seven (now eight) discrete abilities. One of these is called 'logico-mathematical' intelligence, and is to do with logic, inductive and deductive reasoning, and numbers. It is not necessary that this intelligence should correspond closely to mathematical ability, and measures of it have been shown, sometimes, to correlate better with verbal test scores than maths ones. While popular in education, largely because it accords with the traditional notion of intelligence while seeming to include far more pupils as 'intelligent' in different ways, the theory is still controversial amongst academics.

One researcher who has adopted a multiple ability model in a very practical way is Julian Stanley; his programs in Johns Hopkins University have been, since 1971, concerned with the promotion of mathematical and verbal excellence, and other possible abilities and intelligences, or the relationships between them, were just not relevant. Stanley's conception of exceptional ability is quite simple: highly able children are *precocious*, and his first program was named the Study of Mathematically Precocious Youth (since his death in 2005 the two studies have been renamed the Julian C. Stanley Study of Exceptional Talent – SET). Although his rationale for this was originally quite pragmatic, the only explicit psychological theory underlying it was that learning is sequential and developmental, that students learn at different rates, and that their teaching ought to be matched to their readiness (Stanley, Keating, & Fox, 1974). Stanley later justified using precocity as a criterion of giftedness because students doing well on material they had not been taught must have developed their own methods, so that the (for example) SAT-M test "must function far more at an analytic level for the [young students] than it does for high school juniors and seniors" (Benbow & Stanley, 1981). The suggestion that 'aptitude' is best realised through problems targeting analytic reasoning appears many times in the literature.

Nancy Robinson (2005) holds a similar view to Stanley, arguing that the level and pace of educational programmes must be adapted to the capacities and knowledge of individual children (Robinson & Robinson, 1982). She does not approve of enrichment in place of acceleration.

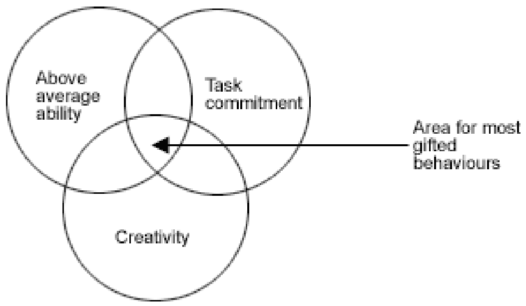
Coleman (eg Coleman, 1985; Coleman & Cross, 2001; Cross & Coleman, 2005) propose a modified version of the definitions within Group 2. In a scheme they call the School-Based Conception of Giftedness they see high quality *performance* as the essential criterion for giftedness, but argue that the effect of schooling is to change in a gradual way how specific each child's giftedness is. For example:

> Preadolescent gifted children have the potential or demonstrated ability in two areas: general cognitive ability and creative ability. Adolescent children have demonstrated ability in abstract thinking, have produced creative works in some worthwhile area, and have demonstrated consistent involvement in activities of either type. (p53)

1.3 a cognitive, affective and motivational combination

Galton did not enquire into the structure of genius, but he did consider its genesis. He proposed, without much research evidence, a tripartite theory of eminence – that it required capacity, zeal, and the power to work hard – a model that is quite general and can be applied to any domain. If we consider 'capacity' to be any specific aptitude then Galton offered an early example of the group of theories that dominated the giftedness field in the 1980s.

Joseph Renzulli proposed his 'Three-Ring' conception of giftedness in 1978, identifying three features that are necessary if giftedness is to develop.



In educational contexts, 'above average ability' will be the most important of the three prerequisites, but in general: "Gifted behaviour consists of thought and action resulting from an interaction among three basic clusters of human traits, above average general and/or specific abilities, high levels of task commitment, and high levels of creativity. Children who manifest *or are capable of developing* an interaction among the three clusters require a wide variety of educational opportunities, resources, and encouragement above and beyond those ordinarily provided through regular instructional programs" (Renzulli, 2005a, p267). Notice that Renzulli emphasises the idea of gifted behaviour rather than gifted children, a distinction that recurs below.

Amabile (1996) offered a model for 'creativity' which is strikingly similar to Renzulli's model for giftedness; her three components are domain-relevant skills, creativity-relevant skills, and task motivation.

Some models in this group explicitly recognise that talent can only develop if the students are given appropriate opportunities, and if society (or their own peer group) value the abilities developed Tannenbaum's (1983) model is perhaps the most explicit:

> There are five factors that have to mesh in order for a child to become truly gifted: (a) superior general intellect, (b) distinctive special aptitudes, (c) the right blending of nonintellective traits, (d) a challenging environment, and (e) the smile of good fortune at crucial periods of life. (p 49)

These models include non-cognitive components such as motivation, persistence and need to achieve alongside intellectual abilities. Robinson (2005) rejects this approach as 'description' rather than 'definition': "these assets are essential, but they are highly dependent on environment and opportunity". In other words, it is the ability itself, and its development, that generate the affective and motivational traits that are observed to accompany high ability. There is recent empirical evidence supporting this: Muijs (1997) noted that "[a] large body of research supports the existence of an academic achievement--self-concept relationship. Path analyses mostly show academic achievement to be causally predominant in this relationship", and confirmed this experimentally for Belgian primary school children: "academic achievement was causally predominant over academic self-concept [which] suggest[s] that self-esteem enhancement in itself cannot be a solution to the problem of academic failure". More specifically, Ma & Xu (2004), using data from the Longitudinal Study of American Youth, showed that maths anxiety is an effect rather than a cause of low achievement.

Without straying too far from the cognitive domain, Sternberg (1985, 1986) described a Triarchic Theory of intellectual giftedness. One aspect concerned cognitive processes and included metacognitive skills alongside general and specific learning skills; a second concerned the ability to relate cognition to experience, to deal with novelty and to automatise information processing; the third concerned applying skills and knowledge, the ability to adapt to, select and shape the environments within which we function. He also provided tests for measuring these abilities (unpublished, 1993; Sternberg et al 2001).

He later incorporated this into the WISC model of giftedness (Sternberg, 2003), an acronym for Wisdom, Intelligence and Creativity, Synthesised. In this model Intelligence comes first; it is a prerequisite for Creativity which involves three sorts of ability – creative, analytic and practical – all of which can be developed within a specific domain; and both are a prerequisite for Wisdom. Wisdom is defined as:

> the application of intelligence and creativity as mediated by values toward the achievement of a common good through a balance among (a) intrapersonal, (b) interpersonal, and (c) extrapersonal interests, over the (a) short- and (b) long-terms to achieve a balance among (a) adaptation to existing environments, (b) shaping of existing environments, and (c) selection of new environments. (Sternberg, 2005, p 334)

Once again, Sternberg (2005) claims that these components can all be measured in individuals. Like Robinson, he argues that motivation is largely situational:

> With the proper environment, anyone can be motivated to achieve. (p 340)

Recognising a need to incorporate more of the external world to explain the emergence or non-emergence of giftedness, Mönks (1992) expanded the Renzulli's Three-ring Conception of Giftedness by including the influences exerted by peers, parents and teachers.

1.4a deliberate practice

The research in groups 4 is somewhat tangential. Rather than full-blown theories of giftedness they represent two sources of evidence that have come to prominence in the last 25 years that theorists of giftedness have consequently drawn on in their own work.

The first source is work on the concept of 'expertise'; the most accessible review of this research is Ericsson et al (2006). The best-known expertise research, and the most influential in theories about giftedness, led to the concept of *deliberate practice*, (Ericsson et al, 1993; Ericsson, 2006). Ericsson argues that talent is less important than practice, that no one will become an expert without many hours of deliberate practice of the requisite skills.

Studies of experts in memory and calculating, in surgery, computer programming, music, chess and other domains showed similar results. For example, serious amateur pianists needed at least 2,000 hours of practice, the least accomplished experts 5,000 hours and the most accomplished 10,000 hours. This has become known as the '10,000 hour rule'. Ericsson also reported that (1) measures of general basic capacities do not predict success in a domain, (2) the superior performance of experts is often very domain specific and transfer outside their narrow area of expertise is surprisingly limited and (3) systematic differences between experts and less proficient individuals nearly always reflect attributes acquired by the experts during their lengthy training (Ericsson & Lehmann, 1996). In this view 'talent' is often used as an excuse: being 'no good at maths' really means 'I don't have the desire to be good at maths and to undertake the hours of deliberate practice it would need to make me better'. There is also a suggestion that such a 'desire' is the most fundamental prerequisite for excellence - or indeed for competence.

Some researchers, especially in Germany, have incorporated the notion of deliberate practice into a model based on prior ability and learning based on other research traditions by proposing the notion of thresholds, levels of ability above which the desire to become excellent can set this positive feedback mechanism going – desire generates improvement which increases a sense of reward which reinforces the desire to succeed.

1.4b genetic and neuroscientific evidence

Prior to 1980 the only sense in which genetics 'contributed' to the debates on ability and excellence was in terms of the heritability of intelligence and specific skills. This was essentially a statistical endeavour to estimate parameters that could not be derived from the biology itself. Since then, it has been recognised that genetic principles established through experimental studies with fruit flies and the like can be applied to understanding human abilities.

Simonton (2005) points out that a trait as complex as mathematical ability will arise from the combination of many genetic factors, and that this combination is likely to be, at least in part, multiplicative rather than additive. Several consequences follow: there will be threshold values since a very low value on *any* component will nullify high values on *all* the others; normally distributed components can combine to generate *very* skewed complex abilities; different combinations of component skill levels can lead to the same resulting overall level of ability, which will make it very difficult to predict giftedness from measurement of the components.

He also notes the relevance of epigenetic principles; specifically, different components will emerge at different times for different people, resulting in more forms of complexity. Some will bloom early, and fade as others catch up on their early relative promise; others will bloom late. The focus of a person's giftedness may shift throughout life, from one specialism to another within the same general domain. And, of course, this again will defeat any attempt to identify future excellence at an early stage.

Neurological research has now more or less established that basic ability to handle numbers is built into our brains:

> The current best guess is that a relatively small part [of the left parietal lobe], the inferior lobule, is the core of our numerical abilities. (Butterworth, 1999)

During the processing of mathematical material scans show high levels of activity in this region, but also in several regions of the prefrontal cortex. (The earliest scans were by Roland & Friborg, 1985.)

Evidence for innate mathematical (ie numerical) ability, however, is not the same as evidence of innate differences. Heritability studies (Vandenburg, 1966) estimated that about half of individual variability in mathematical ability was inherited, but this conclusion has since been widely contested. Twin research is sensitive to many subtle but powerful biological influences (Dehaene, 1997, p158). The same is true, Dehaene argues, of the other key methodology – group and sex differences – and he concludes:

> I am convinced that the prejudices that our societies convey about mathematics are largely responsible for the gap that separates the mathematical scores of men and women, as well as those of rich and poor – a gap that could partially be filled by political and social changes in attitudes to mathematics. (p160)

Butterworth (2006) notes there is "ample evidence for activity-dependent plasticity: that is, that the functioning, and even the structure, of brain systems is shaped by practice and experience" (p565), which provides support for Ericsson's argument on the importance of deliberate practice. [This generally relates to the prefrontal regions noted above.] He concludes by returning to "Galton's tripartite theory of eminence: capacity, zeal, and the ability to do a very great deal of hard work" (p563).

Dehaene describes (pp 91-117) how mathematical ability and even simple numerical skills are not immune to non-mathematical influences. Chinese subjects average about 9 in digit span memory tests, compared to only about 7 for British or American subjects; on average a native Welsh speaking pupil takes 1.5 seconds longer to solve 134 + 88 = ? than an English speaking one. These, and many similar differences around the world, result simply from the way we use verbal names for numbers in carrying out arithmetical operations: the names of digits are shorter in Chinese than in English, and number names are shorter in English than in Welsh. Other examples abound of the influence of language on mathematical processing.

The biological evidence, then, shows that mathematical ability begins as a set of cognitive components which may be inherited, builds on a hard-wired system for handling number which every (normal) person is given by birth, and then develops through a complex process of interaction between these components and between them and external factors, including the person's own other mental activity.

1.5a developmental and synthetic models

From here it should not be surprising that many recent models explicitly address the processes by which giftedness develops. Some of the models already discussed (notably Renzulli's, Robinson's and Stanley) describe themselves as 'developmental' models, meaning that they are based on theories and principles of the psychological development of young people; they do not, however, offer detailed accounts of how giftedness develops.

Benjamin Bloom (1985) reported a series of studies into the components of talent development, and proposed a three stage model, involving (a) a child's recreational involvement with a domain, then (b) a period of explicit teaching and learning about the domain, followed by (c) self-identification as a mathematician/musician or whatever, during which the young person becomes his or her own best critic. In this final phase, if self-doubt is overcome, the person will seek opportunities to learn further, to demonstrate their ability, and to profit from all of this.

Around the same time Françoys Gagné proposed a model with more noncognitive input than Bloom's, which has been revised several times. The current version of the Dynamic Model of Giftedness and Talent can be represented diagrammatically as:

2↓
1 →→ 4
3↑

with the following significance:

1 Natural Abilities (Intellectual, Creative, Socioeffective, Sensorimotor) – also called *Gifts*
2 Intrapersonal (Physical/mental characteristics, Self-management)
3 Environmental (Milieu, Persons, Provisions, Events)
4 Systematically Developed Skills (Various fields) – also called *Talents*. (Gagné, 2003)

Thus Natural Abilities are said to develop into Systematically Developed Skills, under the various influences of Intrapersonal and Environmental factors, people and events. Gagné also explicitly allows Chance to influence blocks 1, 2 and 3 of his scheme – and hence to affect 4. His research over several decades has explored detailed mechanisms within this model.

Heller and colleagues have developed a similar framework in Germany. The Munich Process Model (1992) can be represented by the Gagné diagram with the following significance:

1	talent factors (intellectual, creative, social, practical, artistic, musical, psycho-motor)	(predictors)
2	noncognitive personality characteristics (stress, achievement motivation, strategies, anxiety, control expectations)	(moderators)
3	environmental conditions (intellectual, creative, social, practical, artistic, musical, psycho-motor)	(moderators)
4	performance areas (maths, sciences, technology, computing/chess, art/music, languages, sport, social)	(criteria)

(cited in Heller et al, 2005). This had been developed by 1997 into the Munich Process Model in which each of the four blocks is analysed into the psychological processes that are involved in the factors they represent. Thus, for example, block 4 (the Talents) is re-presented as 'Exceptional achievements: Routine/effectivity; Inventions; Success in negotiation; Ideas for improvement; Leadership/management; And so on (Ziegler & Perleth, 1997). These two models have been merged into what is essentially a synthetic framework for a collection of theories and sub-models from education, developmental and cognitive psychology, and expertise research; labeled the Munich Dynamic Ability-Achievement Model (Perleth, 2001, reproduced in Heller et al (2005)). There is a vast amount of detail in this system, much of it currently under investigation.

From the same earlier theories Albert Ziegler has developed a different perspective on the development of giftedness in his Actiope theory (Ziegler, 2004, 2005). First he argues that the individual is not the "owner" of gifts. Suppose that, overnight, the height of a basketball net is lowered 20cm; many tall and erstwhile 'gifted' players would suddenly 'lose' their gift without themselves changing in any way. Technological innovations like computing and calculators have similarly altered the meaning of 'gifted' in mathematics. This leads him to propose that we focus not on gifted individuals but on the actions they carry out that we see as indicators of giftedness, and explore the genesis of these actions.

He starts by sharpening Ericsson's '10,000' hour rule, by splitting it into Bloom's three stages, and associating different kinds of goals to each. At first the dominant goal is enjoyment; in the second phase it is the improvement that motivates the child, and in the third the appropriate goal is faultless execution of the actions. With Ericsson (1998) he has shown that in the third stage, playing music or chess for pleasure or as a performance, will *not* improve performance – only deliberate practice with the aim of achieving faultlessness will do that. Throughout, the student develops a growing repertoire of useful actions, together with the knowledge, skills and attitudes to exploit them successfully in context: "the focus is no longer on personal attributes, but on actions and their development within a complex system" (Ziegler, 2004, p5).

Feldhusen (2005) presents a similar view, emphasising that students "with high talent potential" need plenty of appropriate opportunities to develop their talents. "Giftedness is a result of nature and nurture". He makes clear the interactional nature of developed talent, arguing that "nature endowed a child with aptitudes, talents, or intelligence above or far above the average child". He thinks the term 'gifts' should be used only for what the family, school and society 'give' the child to nurture these talents (p74).

Van Tassel-Baska (2005) restricts her theory of Domain-Specific Giftedness to "intellective" abilities. In this she is specifically rejecting models like Renzulli's which give prominence to nonintellective traits:

> "It is my contention that motivation, task commitment, and even creativity are born of the talent development process itself and not part of giftedness per se. Hence they are secondary considerations in thinking about the conception of giftedness, rather providing the fuel for the development of aptitude." (p360-1)

Since Gagné, Heller and Ziegler also give prominence to the nonintellective traits it might seem that she is rejecting their models too but, in fact, they all (and Ma & Xu) agree with the contention that creativity and affective traits are as much an effect as a cause of improving levels of achievement.

Subotnik and Jarvin (2005) provide a model which is largely Ericsson's expertise system elaborated. Based on Bloom, and like Heller, they envisage three stages, which they describe as the transformation of abilities into competencies, competency into expertise, and expertise into elite performance. Throughout this they emphasise the possibilities that exist for any student's abilities to be enhanced, and how a high-quality teacher can facilitate each transition.

1.5b context models

Like Ziegler, Plucker and Barab (2005) want to move away from the view of giftedness "residing within the individual" (p 203), criticising this as a modern dualistic fallacy splitting the learner from the context. Reviewing the last twenty years of research in social psychology, learning and thinking styles and other fields they believe "that giftedness is the visible result of the *interaction* of individual and environment" (p 206). They advocate thinking of giftedness only in specific contexts – a situated theory: "Anyone can be talented, yet one needs the opportunity to engage in talented transactions to realize their giftedness" (p 207).

1.6 no model of giftedness

Borland (2005), after many years of working with various models of giftedness, now wishes to dispense with models altogether. In fact, his position is not very far from that of Plucker and Barab, as his main concern is that we should stop imagining 'gifted children'. He believes that programmes to provide special treatment for gifted children have not proved very successful in general, and proposes an alternative aim for education:

> "I am suggesting that we dispense with the concept of giftedness – and such attendant things as definitions, identification procedures, and pull-out programs – and focus instead on the goal of differentiating curricula and instruction for all of the diverse students in our schools." (p12-3)

Rather than seeing most students as 'unexceptional' and a few as 'gifted' he advocates seeing every child as able to engage in 'gifted education'. He would dispense with special provision and instead provide "inclusive schools with differentiated classes, no labeling of students, and differentiated, responsive curricula and instruction" (p14).

In Russia, it seems, Borland's view is common, but it is not seen as antithetical to the selection of specially gifted pupils. Jeltova and Grigorenko (2005) report that:

> Russian psychologists and educators distinguish between identifying gifted children and making a prognosis (prediction) for giftedness." The former implies selecting gifted from among the very high achievers; the latter implies "uncovering giftedness in children and providing them with optimal environmental conditions to elicit, nurture, and harness their gifts. (p178)

1.7 Summary

In recent years models of giftedness do seem to have converged to some degree. Most would concur with the following:

1 they define gifted behaviour rather than gifted students;
2 they emphasise the processes that are crucial to the development of talents;
3 they concentrate on intellective abilities as the starting point;
4 they distinguish about three phases of development to exceptional levels;
5 they conceive of creativity, and affective traits like motivation, as developing out of experience;
6 and recognise the power of motivation and personal goals in encouraging learning;
7 they recognise the influence of many environmental factors in providing and moderating opportunities for talent development;
8 they define giftedness in the behaviour of excellent students/practitioners, rather than in the products of that behaviour.

The differences seem mostly to arise from different interests, and in particular from whether they focus on general school learning or on special abilities like music or chess. They also differ in how they would recommend identifying gifted students.

2 Definitions

"There are perhaps 100 definitions of giftedness around" according to Hany (1987, 1993); "over 180 definitions of giftedness have been put forward" according to NAGC-Britain (undated), currently on their web-site.

2.1 Hany's classification

In fact, it's not really possible to count them, especially if both essentialist and operational definitions are to be included. Hany classifies definitions into four groups, mainly in terms of the kind of theory on which they are based; given the variety of models discussed above it is then inevitable that there will be many and inconsistent definitions. Some important distinctions can, however, be made.

The most traditional approach is based on the concept of *aptitude*, general cognitive ability or abilities that are presumed to predict future success. This is the obligatory approach for anyone who adopts a model of the Group 1 type above where giftedness is a unitary concept. Hany also includes here Group 2 models that propose discrete cognitive abilities. Intelligence tests, aptitude tests and aptitude profiles are offered by many test publishers to support these.

His second group of definitions is labeled the *component* approach, and relate to Group 3 models like Rentzulli's which include non-cognitive or non-intellective components such as creativity and motivation. According to Mönks & Katzko (2005, p 189) "[t]he important theoretical advance is that a more analytic approach is taken in analyzing a multicomponent processing approach".

Many of the models mentioned above (mainly in Groups 4 and 5a) reject the focus on aptitude and insist that giftedness must be defined in relation to real achievement or performance rather than estimates of potential; this is Hany's third group. This leads to identification procedures relying on measures of specific achievement or observations of behaviour.

Some of the more developed Group5a models belong to Hany's final category, with definitions that emphasise the influence and significance of *environmental* factors both in defining and in developing gifts. In the scheme above, models in Groups 5b and 6 also belong here

The view from the Netherlands, and of the German researchers too it seems, is that these four groups represent a historical progression:

> In summary, the research and program provisions literature is currently dominated by descriptive and categorical approaches that fit the first type of definition as well as an emphasis on the third definition fitting an applied approach in educational settings. However, we can expect that, as the analytic approaches become more advanced, reflected in cognitive information-processing models, they will subsume the approaches of the first type. The fourth type of definition, with its emphasis on socio-economic and political concerns, will increasingly affect the availability and planning of programs for the gifted rather than purely theoretical research on the gifted.
>
> Mönks & Katzko (2005, p190)

This continental view may prove too progressive for British and American circumstances.

2.2 Gifts and talents

> I believe that the term *gifted* and the term *talented* have outlived their usefulness. We have little or no consensus about what constitutes these concepts, despite the fact that each of us is willing to write so authoritatively about them.
>
> Robinson (2005, p 282)

Some researchers, like Gagné, explicitly define 'gifts' as the aptitude domains from which 'talents' may develop, but in Tannenbaum's model a child has to "become truly gifted". Robinson notes that the USA's National Association for Gifted Children tried to standardise terminology, but abandoned the task. DfES claims that:

> In England the term 'gifted' refers to those pupils who are capable of excelling in academic subjects such as English or History. 'Talented' refers to those pupils who may excel in areas requiring visio-spatial skills or practical abilities, such as in games and PE, drama, or art. (DfES, 2006a)

But there is little support for this opinion. Simonton refuses to try to separate the terms and, since I am quoting from so many sources in this review, so do I.

2.3 Aptitude and ability

In a commentary on a report from Stanley's SMPY program, Anastasi (1974) commented:

> . . . two different tests of developed mathematical ability were administered [SAT-M and Mathematical Achievement Level 1]. I trust there is no intention to perpetuate the mythological distinction between aptitude and achievement tests – a distinction that not only is conceptually indefensible but also has been empirically disproved by many correlational studies. (p 90)

She always maintained that all tests measure ability, but this ability is neither innate nor immutable: her preferred phrase was always that a maths test measures the current level of developed mathematical ability. Most psychometricians accept this view, but Angoff (1988) argued for the retention of the concept of aptitude for its usefulness, as it lays a different emphasis – on informal rather than formal learning, for instance, or on more general rather than more specific skills. It is still very difficult to give a satisfactory definition of mathematical aptitude in any terms other than merely saying that it is what predicts future mathematical performance reasonably well. A reasonable compromise may be to view an 'aptitude test' as a shorthand term for an achievement test that uses a particular subset of all the possible items, a subset whose members are thought (or shown) to be better predictors of future ability than the others.

2.4 Quantification

Quite apart from differences arising from varying theoretical stances, definitions of giftedness vary in where they set the boundary between being and not being gifted. This problem highlights a curious and troubling fact in giftedness research. In nearly every case involving exceptionally high mathematical ability, and in most cases involving other academic abilities, the boundary is set in a fundamentally normative way; a particular percentage from the top of an ability or performance range is deemed gifted. Terman adopted a 1% criterion for his IQ based definition of genius, Renzulli a 15-20% criterion for his "well above average" Talent Pool (Renzulli, 1986, p 76-7). Gagné (2005, p 109) compromises, by defining 10% as 'mildly gifted', 1% as 'highly gifted', 0.1% as 'highly gifted', 0.01% as 'exceptionally gifted', and 0.001% as 'extremely gifted'.

The problem is that there should be no such difficulty in identifying giftedness, according to most of the models mentioned. Only the aptitude models presume abilities that could be even approximately normally distributed. In Simonton's genetic typology of giftedness only those abilities that are either simple (ie unitary traits like IQ) or 'additive-complex' (ie where multiple components combine additively) can the result be normal. In all other cases, the resulting ability should be skewed or extremely skewed. Yet measures of mathematical achievement generally seem to be distributed fairly normally: is mathematical ability really simple?

In most of the models there is some kind of positive feedback mechanism (eg Ziegler, 2004, p 418), and several also include the idea of thresholds above which development accelerates (eg Renzulli, 2005, p262) or of 'catalysts' (Gagné, 2005, p 105-7); it is an essential feature of Ericsson's theory that achieving motivates more deliberate practice. Any non-linear effect like these should skew the ability distribution significantly, making it easier to identify a distinct sub-group. Why then is it not obvious to everybody which children are very good at maths? Is it possible that most students find some parts of maths interesting enough (though they often won't admit it) to enable the positive feedback effect to improve their achievement a bit and to smooth out the level of developed ability curve?

3 Identification

Many of the distinctions in theories have little effect on identification procedures except to argue for rather different emphasis on, and interpretation of, particular aspects. There are, though, some important distinctions:

3.1 Aptitude or achievement or reasoning or performance

Freeman (1998) states that "high intelligence, as measured by IQ, is by far the most popular criterion for defining children as very able or gifted". From reading many authors, however, the impression is that it is not IQ so much as a profile of cognitive abilities that is used, ranging from the two, verbal and quantitative, identified by Spearman and Vernon to the 21 in the British Ability Scales. Within mathematics, there seems little reason to use a concept of general aptitude, and the argument is between tests of numerical/quantitative aptitude and measures of achievement. Note that spatial reasoning ability is not generally considered useful as a predictor of maths talent or ability (Robinson, 2005, p 286; Krutetskii, 1976, p 351; Gardner, 1983; Anastasi, 1974, p 90).

Many recent theories insist that giftedness should only be assessed on current performance in context, either because giftedness can only be defined in terms of a social context (Tannenbaum, 1983; Freeman, 2005) or because they define 'gifted behaviour' rather than 'gifted children' (Ziegler, 2004; Plucker & Barab, 2005). In Russia, with a highly centralised and standardised system, identification has long been mostly a matter of identifying those who are doing better than others in regular maths achievement; for historical ideological reasons there has always been a reluctance to acknowledge individual differences or the innateness of talents (Jeltova & Grigorenko, 2005, pp 171-2). Krutetskii (1976), however, proposed that "Anyone can become an ordinary mathematician: one must be born an outstanding, talented mathematician" (p361) and cited support for this view from two other eminent Soviets, the mathematician A. N. Kolmogorov and the scientist I. E. Tamm.

3.2 Precocity

Very interesting in this respect is Julian Stanley's view of *precocity*. In the beginning he seems to have adopted his procedure for reasons mostly of convenience (Stanley, 1974), but later justified it in more principled terms. In the Study of Mathematically Precocious Youth (SMPY) the "indicator of mathematical talent is simply a high score at an early age on the mathematics section of the College Board's Scholastic Aptitude Test (SAT-M)" (Stanley & Benbow, 1985, p361). The SAT-M is widely used as a measure of mathematical reasoning ability, and is designed for use with above average students aged about 17-19. SMPY applies it to students in seventh grade, who are aged about 13, and who generally "do not know even first-year algebra well". It is clear, therefore, that these young students must achieve their high scores in ways quite different from those used by the older ones. "Presumably, this could occur only by the use of extraordinary ability at the 'analysis' level of Bloom's (1956) taxonomy" (p362). They also insist that "SMPY has sought already evident ability, rather than some presumed underlying potential that has not yet become manifest" (p362), explicitly denying that the SAT-M acts as a measure of aptitude rather than ability (cf Anastasi and Angoff). The Johns Hopkins programs still run, and the selection procedure is obviously quite successful. The same approach has been used in Singapore (Lim, 1995). It seems obvious that such out-of-stage tests in Britain could only be useful in a low-stakes context, since there is no programme currently in place that generates new mathematical aptitude tests for older pupils on an annual basis and the same test would have to be used repeatedly.

DfES (2006b) appear to distinguish giftedness from precocity (p 6), presumably on the grounds that early appearance of some abilities in very young children may not reliably indicate later giftedness, but Stanley was using the term with adolescents where the criterion is clearly much more reliable. Describing talented maths students as 'precocious' makes it seem that they are just like clever older students, though much younger. This is clearly not how Stanley eventually considered them: what older students could do after being taught the young ones could do without teaching, apparently discovering for themselves a method that leads to a solution. This suggests two possible principles for identifying mathematical talent (they may amount to the same thing):

(1) use tests made up from questions that are appropriate for older children; and

(2) use tests made up from problems that require highly analytic thinking.

The Hamburg 'talent searches' targeted 12 year olds (Wagner & Zimmermann, 1986) and also used SAT-M as a primary selection tool, but accompanied it with seven 'problems' created for the purpose; these involved deriving rules for simple cases and generalising to more complex ones, or inventing a form of analysis or description that helped the student find particular cases. Thus, in accordance with Stanley's explanation for the suitability of SAT-M, the Hamburg group used tasks that demanded very high forms of analytical thinking from students with little or no formal teaching of algebra.

3.3 Question types

It is also clear from the aptitude/ability debate that the nature of the construct being tested by predictive tests should not be the same as in a regular achievement test. An end of key stage test measures how much or how well pupils have learned what they have been taught, and can be used to predict how well they will do in future tests of the same kind if they continue with the same commitment to their studies. But we cannot tell how much commitment went into each pupil's performance, which is essential if we are to distinguish the talented from the merely industrious. This depends on setting questions on which the industrious gain little or no benefit from their industry, the ones that Stanley meant as requiring high analytic thinking.

Niederer et al (2003) studied the use of the Progressive Achievement Test to identify mathematically gifted children in New Zealand, comparing the results to performance on "a published problem-solving test", and concluded:

> the PAT's accuracy at identifying mathematically gifted children was 78 per cent. Such a degree of accuracy will lead to many mathematically gifted children being overlooked, or many being mistakenly identified as gifted. Use of the PAT for this purpose is therefore not recommended.

It seems safe to conclude that:
(3) regular achievement tests are only weak indicators of high talent.

Other researchers agree that the talented are not like older children. Von Károlyi and Winner (2005) write: "Extremely gifted children are different. They develop on a different timetable, their drive is different, they march to a different drummer, and they feel different from others" (p 390). "At all levels of giftedness, children are well aware that they are different. They perceive themselves as different from others and feel that others see and treat them differently (Cross, Coleman, & Stewart, 1993; Freeman, 1994; Janos & Robinson, 1985; Janos, Fung, & Robinson, 1974: Robinson, 1990; Subotnik, Kassan, Summers, & Wasser, 1993)."

The gifted are torn between two referential populations, those of the same ability and those of the same age. To paraphrase LP Hartley, for gifted children: "the 'average' is a foreign country; they do things differently there."

Some researchers believe that no standardised tests are very useful in identifying mathematical talent. Runco (2005) argues for the critical importance of creativity in any form of giftedness, and believes the "capacity for creative performance is widely distributed" (p 298). "To understand is to invent", he writes, a view which is consistent with the mental model/representation theories of general cognition (Johnson-Laird, 1983) and of reading and listening comprehension (Gernsbacher, 1990), and which suggest that almost any student could become gifted in some field. For others, like Plucker & Barab, or Van Tassel-Baska as discussed earlier, tests are simply inappropriate because they are decontextualised and inauthentic.

3.4 The stigma of giftedness

The Cross, Coleman & Stewart article mentioned above refers in its title to "the stigma of giftedness paradigm", more of a syndrome in which students are reluctant to admit to their high ability or to let other students or their teachers notice it. In Victoria, Australia, the official advice on identifying 'gifted and high potential students' makes the point very early on that

> Some students who have high potential may mask their potential from teachers by not achieving in the upper ranges of the class or by engaging in generally disruptive or low-motivation behaviours.
>
> (Victoria, 2003)

Koshy (2001) gives some examples of the problem: talented pupils "may not want to be different"; they may not want to be "given extra pages to do or harder versions of the same type of work"; a child explained "I will lose all my friends . . . If I give them the answers my teacher gets mad at me and if I don't the other children will beat me up".

Damarin (2000) notes how, in our culture, those who are known to be 'mathematically able" constitute what sociologists call a *marked category*, meaning that they are liable to be maligned, ridiculed, and discriminated against, even feared in some circumstances because of the power they can wield despite being marked as powerless. Women, too, are a marked category, and therefore girls who are good at maths are doubly marked, open to serious discrimination and firmly marginalised. She asks, rhetorically, "How many young women leave the study of mathematics?" because they are doubly marked in this way. The same could be said for pupils belonging to various other social sub-groups.

It is worth noting that this social acceptability of self-proclaimed mathematical incompetence is not universal. Freeman (2005) notes that Scandinavian countries seem unconcerned with the concept of giftedness, while the influence of Confucian views in much of eastern Asia means that all children are assumed to begin with similar abilities; there success is considered to come from hard work and good teaching – here success is largely considered a matter of luck in being born with or without mathematical (or other) potential.

Whatever the reasons, the social pressure to hide mathematical ability is a serious problem for any identification system. On a Western view, there are many children whose mathematical light is more or less deliberately hidden. Combining this concern with the epigenetic view that talents may emerge at any time, and the variously expressed opinion that contingencies may trigger such emergence (critical life events, chance, interest, etc), the obvious conclusion is that children should encounter many opportunities to be identified, using a very wide range of criteria. To meet particular concerns with likely under-representation of students from minority communities, from low socioeconomic status environments, or with limited English speaking ability, some authors recommend *authentic assessments*: "performance tasks that are part of the child's world—problems that a child can see as real problems and potential solutions to real challenges . . . have greater face validity for children who are threatened by, turned off by, or otherwise inadequately assessed by paper-and-pencil intelligence, aptitude, or achievement tests that contain items not related to their everyday existence or to the kinds of performance that has been common for the majority child" (Callahan, 2005, p 101). In England, Sumner (1987) commented that achievement tests are as good as anything else at predicting future achievement, but also noted that 'suppressed talent' can't be detected by the usual attainment tests, and could only be spotted, perhaps, using tasks that do not resemble school tests.

These considerations suggest:

(4) assess frequently, using many formal and informal kinds of evidence; and

(5) use assessment tasks that have high authenticity and face validity.

While it is obviously desirable to identify 'hidden talent', a note of caution may be necessary: if the identification is to admit students into a special programme, it's important that they should be able to benefit from it. Jones & Byrnes (2006) showed that "both opportunity and propensity are important (i.e., students must be *willing* and *able* to take advantage of opportunity)". Key predictors of success in a special algebra program were cognitive (prior knowledge), affective (not suffering high frustration) and metacognitive (self-regulation). Students unearthed by careful identification procedures may need careful nurturing to help them acquire each of these characteristics well enough to avoid failure.

3.5 Identification schemes

Amongst researchers at least, there seems to be substantial agreement about the nature of a good scheme for identifying extreme mathematical talent; most want several sources of information to be considered, and at least two stages in the process. One strategic difference is between a ‘subtractive’ process and an ‘additive’ one.

In his ERIC digest, Miller (1990) recommended the first approach. A first phase is used to “establish a group of students **suspected** of having high ability in mathematics” (my emphasis). A list of students indicated by the results of aptitude tests is made, and to it are added other students scoring high on achievement tests, other students showing indications of creativity, special interest in maths, and any others identified using a checklist of behavioural characteristics, or nominated by parents, teachers, students themselves or peers.

This pool of the potentially gifted is then **reduced**. An out-of-grade-level test is used to separate those who are merely “good students” from the genuinely talented, after a prior review to remove any students for whom the use of such a test might be inappropriate. “Generally, the student's out-of-grade-level score will be an indication of degree of mathematical talent.”

The most recent proposal in Britain uses the second, additive, approach. McClure & Piggott (2007) give advice to teachers to help them meet the current expectation that a school will identify about 10% of its pupils as gifted or talented. In this context they advise beginning with a list of “the students that **everyone agrees** are highly able and high achieving” (again my emphasis), and confirming these against a checklist.

This minimal pool of the gifted is then **increased**. A checklist, combined with interviews and nominations from parents and peers and other sources should then be used to add further names, of those who are talented but underachieving. More may be added if aptitude test scores are available. In line with national advice, they next advise teachers to consider how this list reflects the make up of the whole cohort in terms of gender and other identifiable subgroups.

Both Miller and McClure & Piggott urge their readers to keep the identification under review, and to look repeatedly for new evidence that might indicate special ability amongst their students. The procedures of identification and communication with students and their parents need to be handled with care.

The Heller and Ziegler tradition in Germany has led to a model called ENTER which seems to be subtractive in nature (Ziegler & Stöger, 2003), The letters are an acronym for explore, narrow, test, evaluate and review; in the ‘ENT’ phases many kinds of biographic and assessment evidence are used to fill out all of the components of a multifactor model of the development of giftedness, all of which then inform the ‘Evaluation’ phase. It appears that the process results in a set of proposals for each student, but a detailed description of the model has yet to appear in English (Ziegler & Stöger, in press).

3.6 Checklists

There are many checklists available that indicate the kinds of behaviours and abilities that are typical of the mathematically exceptionally able – eg McClure & Piggott (2007). Kennard (2001), QCA (2001), Sheffield (1994). All of these owe their origin to Krutetskii’s lengthy programme of investigations into the nature of mathematical abilities in Russian schoolchildren (Krutetskii, 1976); his empirical studies are important particularly for showing which skills are indicative of high mathematical potential (such as generalising, using symbols and curtailment of thinking) and which are not (such as swiftness, computational ability and spatial ability).

The wide use of checklists indicates that they are useful in identification, but some writers give warnings that they may be misused. Koshy (2001) gives reasons why it would be “unwise” to use them as the sole means of identification without considering fully the context in which the pupils are developing and the social or cultural handicaps that might affect them. Callahan (2005) warns against the use of a matrix, used in some American systems, where each point in a checklist of possible indicators is given an arbitrary score, and the total is supposed to indicate talent; there is always a risk that a checklist or tick-counting approach will encourage this kind of automatic behaviour in place of real consideration of each child as an individual.

3.7 Non-traditional assessment

Van Tassel-Baska (2005) suggested that some non-traditional forms of assessment are needed to improve the identification of what she describes as ‘domain-specific giftedness’. These include nomination by parents and peers, and creativity tests, performance-based assessment and portfolios.

Dynamic assessment has also been proposed (Kirschenbaum, 1998). In this procedure, which is based on Vygotsky’s (eg 1978) notion of the ‘zone of proximal development’, a sequence of test-intervene-retest allows assessment of the improvement students make as a result of cognitive skills and strategies they develop from the experience of the tasks; according to the theory an individual's responsiveness to assistance is indicative of his/her future performance.

3.8 Gifted education with no gifted students

Finally, it was noted that Borland (2005) advocates abolishing the concept of the ‘gifted student’, and instead aiming to develop the gifts of every student. There are, in fact, several other straws blowing in this direction. Subotnik & Jarvin (2005) emphasised the possibilities that exist for any student’s abilities to be enhanced, with appropriate provision, and discussed how a high-quality teacher can facilitate this. Freeman (2005, p89), speaking of out-of-school programmes, stated that “The growing trend around the world is to offer nonselective open access to very high-level learning opportunities, so that no keen youngster is turned away without even a chance of attempting it” (2005, p89). And Renzulli (2005b) argues that programmes for gifted students have been the source of ”many of the innovations that have subsequently become mainstays of general education in American public schools” (p 80); and that it is time now to extend the methods developed there to all students.

4 Conclusions and recommendations

Existence of MEA in maths

Does a distinctive sub-cohort of mathematically exceptionally able students exist within the GCSE mathematics cohort? It seems strange that there is not an easily identifiable 5 or 10 or 20% group, given the importance given to positive feedback in most models. Perhaps the answer is to stop viewing mathematics as a single achievement domain and mathematical ability as a unitary trait.
Gardner (1995) made this comment:

> While it is notable that some youngsters can achieve a very high score on a mathematics examination like the SAT, this may not be the best marker for who will ultimately make important mathematical contributions. It might be that the future mathematician of distinction stands out because of her interest in knots, her ability to make up new games, or her curiosity about the history of mathematics.
>
> For such reasons, I favor mechanisms that allow individuals to show what it is that they can already accomplish in a domain and to indicate what it is that they would like to accomplish, particularly if they were given additional (and often precious) resources.

If there are many routes to success in maths there must be many feedback effects, and for many students just a few will currently be activated. The key to maximising mathematical success would then be to identify every child who has any interest in anything mathematical, and to try use it to stimulate a wider interest in the subject – perhaps 'simply' by making them realise that they actually are interested in maths.

How can we ensure that none of 'Gardner's youngsters' will fail to have their potentially relevant interests or creativity recorded; some thought should be given to the details of this procedure of the creative identification of real world mathematical interests. One important point would seem to be to adopt the additive concept of identification, as described by McClure and Piggott; only with a positive intent always to look for another child with another possible trigger-point for the development of maths abilities can we hope to find Gardner's youngsters.

It may be that the 'explore' phase of Ziegler & Stöger's ENTER model will provide ideas for this: perhaps in their model the 'explore' phase should never stop? Ziegler should be contacted to see if an English translation of the paper is now available.

Recommendation 1: *Spread the idea of an additive search for talent in this specific sense.*

Recommendation 2: *Look at the experiences of the German and Dutch teams in the area of identification.*

There is little point in identifying pupils for special treatment if they are unwilling to participate positively. Indeed, one of the criteria for inclusion is often 'showing an interest' – though this does need to be interpreted rather rigorously. But an argument can be made for allowing pupils to nominate themselves for any such program. (Freeman, 2005, quoted above) described a trend to allow open access to out of school provision, at least for a trial period. Why should a positive feedback loop only be entered at the 'high attainment' point and not also at the 'high interest' point?

This also suggests that the 'best fit' approach advised by DfES (2006b) may give a misleading impression. If Gardner is right a "future mathematician of distinction" may not fit most of the criteria in a checklist of characteristics of gifted mathematics pupils. If maths ability or interest is not unitary then a teacher should fasten onto any single criterion they spot as a possible starting point, even when the pupil otherwise looks very unpromising.

Recommendation 3: *Compile a list, from teachers' experiences, of individual cases in which an isolated interest of the kind Gardner described has provided a starting point.*

This recommendation should help all teachers motivate all pupils, and is not limited to a small sub-cohort of extremely mathematically able children.

Recommendation 4: Change 'best fit' model to 'any indication' model.

It is unlikely that we could completely remove the dependence on tests in identification, and Gardner was not suggesting they should be abandoned. But Miller's recommendation that tests of aptitude be given priority over tests of attainment is important if we want to identify the potentially extremely able rather than just the hard working and competent.

This review has identified some principles that many experts advocate:

- use tests made up from questions that are appropriate for older children;
- testing should use questions/tasks that target higher analytic reasoning;
- use assessment tasks that have high authenticity and face validity;
- use exploratory tasks, collaborative tasks, etc – anything that seems less like classwork to try to identify hidden talent.

It does not seem right that, as Freeman (1998) wrote, "high intelligence, as measured by IQ, is by far the most popular criterion for defining children as very able or gifted". Even if IQ were replaced by something like the non-verbal component of the Cognitive Abilities Test, this would be far from the advice that is so widely given.

There is plenty of possible material around in Britain that could meet the criteria, perhaps from the World Class Tests or from the NRICH project. Computers open up a potential route for presenting maths problems in ways that will overcome the problems of the traditional test format.

Recommendation 5: Develop a pilot "untestlike test" meeting these criteria to see how it works as an identification tool.

There seems to be enough, and good enough, advice for teachers and others to help them identify exceptional mathematical ability.

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